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Birzeit University
Math. & Comp. Science Dept.
Linear Algebra

Dr. Ragheb Abu-Sarris, Dr. Allaeddin Elayyan, and Dr. Mohammad Saleh

Math 234 Final Exam

First Semester Number:______ Section: ___

Student Name:____________________________________

(Question One: 40 points) Circle the MOST correct statement

1. If the coefficient matrix of a system of linear equations is

$$\begin{pmatrix} 1 & 2 & 3 \\ 0 & 1 & 2 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{pmatrix}$$

then

(a) the system is consistent.
(b) if the system has a solution, it is unique.
(c) if the system has a solution, it has infinitely many solutions.
(d) the system is inconsistent.

2. If $A = (\vec{a}_1, \dots, \vec{a}_n) \in R^{m\times n}$ where $\vec{a}_j$ denotes the j^{th} column of A, then $A\vec{x} = \vec{b}$ is consistent if and only if

(a) the system has a solution.
(b) $\vec{b} \in \text{Span}(\vec{a}_1, \dots, \vec{a}_n)$.
(c) $\text{rank}(A|\vec{b}) = \text{rank}(A)$.
(d) all of the above.

3. If $A \in R^{n\times n}$ such that $A^n = I$, then

(a) $A^{-1} = A^{n-1}$.
(b) ~~$\det(A) = 1$.~~
(c) any $\vec{x} \in R^n$ is an eigenvector of A^n.
(d) all of the above.

$\lambda = 1, 1, 1, 1$

$(A^n - \lambda^n I)X = 0$

$(I - I)X = 0$

$0 \cdot X = 0$

$I - I = 0$ $rX = 0$

40

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(d) all of the above.

4. Let $A = \frac{1}{4}\begin{bmatrix} 4 & -2\sqrt{3} & 0 \\ \sqrt{12} & 4 & 0 \\ 0 & 0 & 4 \end{bmatrix}$.

$$A = \frac{1}{4}\begin{bmatrix} 2 & -2\sqrt{3} & 0 \\ \sqrt{12} & 2 & 0 \\ 0 & 0 & 4 \end{bmatrix}$$

(a) Is A non-singular? explain.

(b) If A^{-1} exists , Find A^{-1}.

(c) Find the $rnak(A)$.

$$\left(\frac{\sqrt{3}}{2}\right)\left(\frac{\sqrt{2}}{4}\right) = \frac{6}{8}$$

$$= \left(\frac{\sqrt{36}}{8}\right)$$

5. Let $A = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 2 & 1 & 0 & 0 \\ 3 & 4 & 2 & 0 \\ 0 & 0 & 5 & 3 \end{bmatrix}$

$$\frac{1}{4} \times \frac{6}{8} = \frac{8 + 24}{32} = 1$$

(a) Find $det(A)$.

(b) Find $C(X)$.

(c) Find $N(A)$ and dimension of $N(A)$.

N.S.M.

Birzeit University

Math. & Comp. Science Dept.

Final Math. 2341 Fall

Dr. Mohammad Saleh

Name:

Q1:(20 Points) True or false:

(a) If A, B are invertible matrices then AB is invertible.

(b) A homogeneous system might have finitely many solutions.

(c) If the system $AX = B$ has more than one solution then A is invertible.

(d) If A, B are two matrices and AB is invertible then A and B are invertible.

(e) If A, B are two square nonzero matrices and $AB = 0$ then A and B are not invertible.

(f) Let A be a square and invertible $n \times n$ matrix. Then $|adj A| = |A|^n$.

(g) Let A be a square $n \times n$ matrix. Then $|3A| = 3^n|A|$.

(h) If A is a symmetric and skew symmetric then A must be a zero matrix.

(i) If A, B are symmetric then AB is symmetric.

(j) If all entries of the main diagonal of a square martix A are zeros then A is not invertible.

(k) every metric space is a normed space.

(l) every normed space is an inner product space.

(m) if x_0 is a solution of the nonhomogeneous system $AX = B$ and x is a solution of the homogeneous system $AX = 0$. Then $x + x_0$ is a solution of the nonhomogeneous system $AX = B$.

(n) If Wronskain$(f_1, \ldots, f_n) = 0$ then $f_1, \ldots, f_n$ are linearly dependent.

(o) If A, B are similar then $p_A(\lambda) = p_B(\lambda)$.

(p) If A is $n \times n$ and diagonalizable, then A has n different eigenvalues.

(q) If 0 is an eigenvalue of A then A is not invertible.

(r) If A is diagonalizable then A is diagonal.

(s) If A is $n \times n$ and has n linearly independent eigenvectors, then A is invertible.

(t) If A is 2×2, $tr(A) = 5$, and 2 is an eigenvalue of A, then 3 is an eigenvalue of A.

Q2 :(22points) Circle the most correct answer

(1) Let A be invertible. Then

(a) if A is symmetric then A^{-1} is symmetric
(b) If A is triangular then A^{-1} is triangular
(c) If A is diagonalizable then A^{-1} is diagonalizable
(d) All of the above

(2) If u, v are orthogonal vectors then

(a) $\|u+v\| = \|u\| + \|v\|$
(b) $\|u\| \cdot \|v\| = \| < u, v > \|$
(c) $\| < u, v > \| = 1$
(d) none

(3) Define $< f, g > = \int_0^1 f(x)g(x)dx$ for every $f, g \in C[0,1]$, then $\|x\| =$

(a) $\frac{1}{2}$
(b) $\frac{1}{\sqrt{2}}$
(c) $\frac{1}{\sqrt{3}}$
(d) $\frac{1}{4}$

(4) One of the following sets of vectors is l.d.

(a) $< 1, 1, 1 >, < 2, 2,$ [illegible]
(b) $< 1, 1, 3 >, < 2,$ [illegible] $4 >, < 1, 0, 0 >$
(c) $1, x, x^2$
(d) $< 1, 0, 1 >, < 2, 3, 5 >, < 6, 4, 3 >, < 3, 2, 5 >, < 5, 0, 0 >$

(5) One of the following is not a basis for the corresponding space

(a) $< 1, 1, 1 >, < 2, 3, 1 >, R^2$
(b) $< 1, 0, 1 >, < 2, 3, 5 >, < 6, 4, 3 >, < 3, 2, 5 >, < 5, 0, 0 >, R^3$
(c) x, x^2, x^3, P_3
(d) *all of the above*

(6) The dimension of the column space of $A = \begin{bmatrix} 1 & 2 & 3 & 0 \\ 0 & -1 & 1 & 0 \\ 2 & 4 & 0 & 1 \end{bmatrix}$ is

(a) 3
(b) 4
(c) 2
(d) 1

(7) If A is $n \times n$ invertible matrix then

(a) $Rank(A) = n$.
(b) $Nullity(A) = n$
(c) $Nullity(A) = n - 1$
(d) $Nullity(A) = n^2$

(8) If an $n \times n$ matrix A has only one eigenvalue λ then dimension of the eigenspace corresponding to λ is.

(a) n.

(b) ≥ 1.

(c) $= 1$.

(d) 0

(9) If an 4×4 matrix A has 1,-1,3,5 as its eigenvalues then $det(A) =$

(a) 8

(b) 15

(c) -15

(d) -8

(10) Let $A = \begin{bmatrix} 1 & 3 \\ 0 & 2 \end{bmatrix}$ then the eigenvalues of A^{100}

(a) 1, 2

(b) $1, 2^{100}$

(c) 2^{100}

(d) *none*

(11) The nullity of $[1\ 1\ 1\ 1]$ is

(a) 1

(b) 2

(c) 3

(d) 4

Q5 :(20points) Let A =

$$\begin{pmatrix} 1 & 1 & 2 \\ 0 & 2 & 1 \\ 0 & 0 & 3 \end{pmatrix}$$

a) Find the eigenvalues and eigenvectors of A.

b) Is A diagonalizable, if yes find a matrix P that diagonalize A.

MATH. 234, FINAL EXAM, FIRST SEMESTER

Dr. Ayman Abuhijleh (Sections 1 and 4)
Dr. Jawad Abuhlail (Section 5)
Dr. Alaeddin Elayyan (sections 2 and 3)
Dr. Hasan Yousef (Section 6)

Name————————, ID. Number———————————,Score

QUESTION 1. (points) (Write Down True or False)

(1) If A and B a $n \times n$ matrices, then $det(AB) = det(A)det(B)$ ( )

(2) If A and B $n \times n$ nonsingular matrices, then $A + B$ is a nonsingular matrix ( )

(3) If A is an n n matrix and $b \in R^n$ such that $AX = b$ has no solution, then A is singula ( )

(4) If A and B are $n \times n$ matrices and A is row equivalent to B, then $det(A) = det(B)$ ( )

(5) It is possible that $x^2 - x$ be the characteristic polynomial of a nonsingular 2×2 matrix.

(6) If A is an $n \times n$ matrix and $det(A) = 0$, then $AX = 0$ has infinitely many solutions

(7) It is possible to have $v_1, v_2, v_3 \in R^4$ such that $span\{v_1, v_2, v_3\} = R^4$ ( )

(8) Let V be the vector space of all 2×3 matrices. Then $dim(V) = 5$. ( )

(9) If A is a 2×2 nonzero singular matrix, then $Nullity(A) = 1$ ( )

(10) If A and B are $n \times n$ matrices and A is row equivalent to B, then A is singular if and only if B is singular ( )

(11) If A is an $n \times n$ matrix and a, b are eigenvalues of A, then $a + b$ is an eigenvalue of A. (———)

(12) If A is an $n \times n$ matrix and a, b are two distinct eigenvalus of A and $v \in Nul(A - aI) \cap Nul(A - bI)$, then v is the zero vector. (———)

(13) If A is a singular $n \times n$ matrix, then 0 is an eigenvalue of A. (———)

(14) If T is a linear transformation from R^4 to R^3, then there is a matrix A, 3×4, such that $T(v) = Av$ for every $v \in R^4$. (———).

(15) There is a linear transformation from R^4 to R^2 such that T is one to one. (———)

(16) If A is an $n \times n$ matrix such that $det(A) = det(-A)$, then n must be an even positive integer. (———)

(17) *If A, B are $n \times n$ nonzero matrices such that AB is the zero matrix, then both A, B are singular.* (———)

(18) *If A is a nonzero 3×2 matrix such that $Ax = 0$ has infinitely many solutions, then $Nullity(A) = 2$.* (———)

(19) *If A is 5×8 matrix such that $Rank(A) = 3$, then $Nullity(A) = 2$.* (———)

(20) *There is a matrix, say A, 6×9 such that $Rank(A) = 7$.* (———)

(21) *If A is a nonsingular $n \times n$ matrix such that $Av = 3v$ for some nonzero vector $v \in R^n$, then $A^{-1}v = -3v$.* (———)

(22) *If A is an 7×4 matrix and $Rank(A) = 4$ and AB is the zeo matrix for some matrix B, 4×2, then B must be the zero matrix.* (———)

(23) *If A, B are nonsingular $n \times n$ matrices, then A is row equivalent to B.* (———)

(24) *If A, B are $n \times n$ matrices and $B = 3A$, then $det(B) = 3det(A)$* (———)

(25) *If A, B are $n \times n$ matrices and A is row equivalent to B, then $A = PB$ for some nonsingular $n \times n$ matrix P.* (———)

(26) $\{x^2+1, x^2+x\}$ *is a basis of* P_3. (———)

(27) $dim(\{x^3+3, x^3+1, x^3+x\}) = 2$. (———)

(28) $S = \{f(x) \in P_3 : f(1) = 0 \text{ or } f(0) = -1\}$ *is a subspace of* P_3. (———)

(29) $S = \{f(x) \in P_5 : f(x) = -f(x)\}$ *is a 3-dimensional subspace of* P_5. (———)

(30) *Given* $S = \{f(x) \in P5 : f(1) = f(-1)\}$ *is a subspace of* P_5. *Then* $\{1, x^2, x^4\}$ *is a basis of* S. (———)

QUESTION 2. *(30 points,* **Each = 1.5 points***)***(CIRCLE THE CORRECT ANSWER***)*

(1) *Let A and B be 3×3 matrices such that $det(A) = -1$ and $det(B) = -2$. Then $det(2AB^{-1}) =$*
(a) *1* (b) *4* (c) *16* (d) *None*

(2) *Given $S = \{A \in R^{4\times 4} : A$ is a diagonal matrix$\}$ is a subspace of $R^{4\times 4}$. Then $dim(S) =$*
(a) *2* (b *3* (c) *4* (d) *None*

(3) *Given $S = \{f(x) \in P_9 : f(1) = f(-2) = 0\}$ is a subspace of P_9. Then $dim(S) =$*
(a) *7* (b) *8* (c) *9* (d) *None*

(4) *If A is a 5×3 matrix and $b \in R^5$ such that $AX = b$ has exactly one solution. Then $Nullity(A) =$*
(a) *0* (b) *at least one* (c) *3* (d) *None*

(5) *Let* $A = \begin{bmatrix} 0 & 0 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}$. *Then the eigenvalues of A are*
(a) *1 and -1* (b) *0, 1, and -1* (c) *-1, 2, and 0* (d) *None*

(6) *If A is a 4×4 matrix and A is similar to* $\begin{bmatrix} 1 & 2 & 0 & 8 \\ 0 & 2 & 1 & -4 \\ 0 & 0 & 1 & 5 \\ 0 & 0 & 0 & 2 \end{bmatrix}$, *then the characteristic polynomial of A is*
(a) $(x-1)^2(x-2)^2$ (b) $(x+1)^2(x+2)^2$ (c) $(x-1)(x-2)$ (d) None

(7) *If -2 is an eigenvalue of a 4×4 matrix A, then an eigenvalue of $4A$ is*
(a) *1/2* (b) *-1/2* (c) *-8* (d) *16*

(8) *Let* $A = \begin{bmatrix} 1 & -1 \\ 1 & 1 \end{bmatrix}$, $B = \begin{bmatrix} -2 & 2 \\ -2 & -2 \end{bmatrix}$, *and* $C = \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix}$. *Then a basis for* $Span\{A, B, C\}$ *is*
(a) $\{A, C\}$ (b) $\{A, B, C\}$ (c) $\{A, B\}$ (d) None

(9) *If v_1, v_2 are independent in R^3 and v_3 is a (nonzero) element of R^3 such that $v_3 = 2v_1 + -3v_2$. Then*
(a) *dim* $Span\{v_1, v_2, v_3\} = 2$ (b) $\{v_1, v_3\}$ *are independent* (c) $\{v_2, v_3\}$ *are independent* (d) *a, b, and c are correct statements*

(10) *ONE of the the following is not a vector space*
(a) $V = \{f(x) \in P_9 : f(0) = 0\}$ (b) $V = span\{e^x, \sin x, \tan x\}$ (c) $V =$ *all upper triangular* 5×5 *matrices* (d) $V = \{f(x) \in P_{13} : f(0) = 1\}$

(11) *Given* $A = \begin{bmatrix} 2 & -3 \\ 2 & -5 \end{bmatrix}$ *is similar to a diagonal matrix D. Then D =*
(a) $\begin{bmatrix} 1 & 0 \\ 0 & -4 \end{bmatrix}$ (b) $\begin{bmatrix} 2 & 0 \\ 0 & -5 \end{bmatrix}$ (c) $\begin{bmatrix} -2 & 0 \\ 0 & 5 \end{bmatrix}$ (d) $\begin{bmatrix} -1 & 0 \\ 0 & 4 \end{bmatrix}$

(12) *Given A is a 2×2 matrix and 2, 1 are eigenvalues of A. Then the characteristic polynomial of A^{-1} is*
(a) $x^2-(3/2)x+1/2$ (b) $x^2+(1/3)x+1/2$ (c) x^2-3x+2 (d) $1/(x^2-3x+2)$

(13) *Let* $T : R^3 \Longrightarrow R^2$ *such that* $T(x_1, x_2, x_3) = (x_1 + x_2, x_2 + x_3)$. *Given* $B = \{(1,2,1), (0,2,-1), (-1,-2,3)\}$ *is a basis for* R^3 *and* $H = \{e_1, e_2\}$ *is the standard basis for* R^2. *Then the matrix representation of T with respect to B and H is*
(a) $\begin{bmatrix} 1 & 0 & -1 \\ 2 & 2 & -2 \\ 1 & -1 & 3 \end{bmatrix}$ (b) $\begin{bmatrix} 3 & 2 & -3 \\ 3 & 1 & 1 \end{bmatrix}$ (c) $\begin{bmatrix} 3 & 3 & 2 \\ 1 & 1 & -3 \end{bmatrix}$ (d) None

(14) *Given A is a $4 \times m$ matrix, B is a $m \times 4$ matrix, and $C = AB$. Suppose that C is nonsingular (observe that C is a 4×43 matrix). Then*
(a) $m < 4$ (b) $m = 4$ (c) $m < 4$ (d) $m \geq 4$.

(15) *Let* $A = \begin{bmatrix} 2 & 0 & 0 & -2 \\ -4 & 0 & 0 & 4 \\ 0 & 3 & 0 & 0 \end{bmatrix}$. *A basis for the row space of A is*
(a) $\{(2,0,0,-2),(-4,0,0,4),(0,3,0,0)\}$ (b) $\{(2,0,0,-2),(0,0,1,0)\}$
(c) $\{(1,0,0,-1),(0,1,1,0\}$ (d) $\{(2,0,0,-2),(0,3,0,0)\}$

(16) *Let* $A = \begin{bmatrix} 1 & 1 & 1 & 0 \\ 2 & 1 & 0 & 1 \end{bmatrix}$. *A basis for N(A) is*
(a) $\{(1,0,-1,1),(0,1,2,-1)\}$ (b) $\{(1,0),(0,1)\}$ (c) $\{(0,-2,1,1),(1,-3,1,0)\}$
(d) $\{(1,-2,1,0),(-1,1,0,1)\}$

(17) *The solutions for*
$$x_1 + x_2 + x_3 + x_4 + x_5 = 2$$
$$x_1 + x_2 + x_3 + 2x_4 + 2x_5 = 3$$
$$x_1 + x_2 + x_3 + 2x_4 + 3x_5 = 2$$
(a) $\{(x_1, x_2, x_3, 2x_1, -x3) : x_1, x_2, x_3 \in R\}$ (b) $\{(2,-1,1+x_3,x_3,0,-2) : x_3 \in R\}$
(c) $\{(1-x_2-x_3, x_2, x_3, 2, -1) : x_2, x_3 \in R\}$ (d) $\{(x_2+x_3, x_2, x_3, 2x_2, -x_3) : x_2, x_3 \in R\}$

(18) *Let* $A = \begin{bmatrix} 1 & 3 & 2 \\ 0 & 0 & (c-3) \\ 0 & 3 & 10 \end{bmatrix}$. *The values of c that make* $AX = b$ *has a solution for every* $b \in R^3$ *are*
(a) *All real numbers* (b) *All real numbers except 3* (c) *0* (d) *All real numbers except 0.*

(19) *Given* $A = \begin{bmatrix} 2 & 4 & 1 \\ 0 & 0 & 2 \\ 0 & -1 & 2 \end{bmatrix}$. *The (1, 3)-entry of* A^{-1} *is*
(a) -2 (b) 2 (c) *0* (d) *None*

(20) *Given* v_1, v_2 *are independent in* R^3. *Then*
(a) *If* $v_3 \in span\{v_1, v_2\}$, *then it is possible that* $\{v_1, v_2, v_3\}$ *form a basis for* R^3 (b) *It is possible that* $(0,0,0) \notin Span\{v_1, v_2\}$
(c) *If Span* $\{v_1, v_2, v_3\} = R^3$, *then* $v_3 \notin Span\{v_1, v_2\}$ (d) *a, b, and c are correct statements*

Dr. Ayman Abu- Hijleh

Final Exam

Summer

Student Name: Badee' Awwad ---------------- Number:

Section: ----

1. Let $A = \cancel{\begin{bmatrix} 1 & 3 \\ 0 & -1 \end{bmatrix}}$. $\quad A = \begin{bmatrix} 2 & -3 \\ 1 & -2 \end{bmatrix}$

   (a) Find the eigenvalues of A.
   (b) Explain, why A is diagonalizable.
   (c) Find a diagonal matrix D and a matrix Q s.t. $Q^{-1}AQ = D$.
   (d) Find A^{103}

2. Let $T : R^2 \to R^3$, given by $T\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} 3x_1 \\ 0 \\ 2x_2 \end{pmatrix}$ is a L.T. Also, given $B_2 = \{(3,2),(1,6)\}$ is a basis for R^2, and $B_3 = \{(2,0,4),(-4,0,2),(0,30)\}$ is a basis for R^3.

   (a) Find the matrix A representing T in terms of B_3 and B_2.
   (b) Find $\left[T\begin{pmatrix} 5 \\ 10 \end{pmatrix}\right]_{B_3}$.
   (c) Find the dimension of $Range(T)$ and the dimension of $Ker(T)$.

3. Let $B = \{10, 3x + 1, 5x^2 + 2x - 1\}$

   (a) Show that B is a basis for P_2.
   (b) Find the transition matrix from B to $\{1, x, x^2\}$.
   (c) Find the transition matrix from $\{1, x, x^2\}$ to B.
   (d) Write $P(x) = 3x^2 + 17x + 33$ as a linear combination of the elements in B.

3

27
17
100
85
15

27
108
8
28

4. If A and B are two matrices such that $(AB)^t = A^tB^t$, then

(a) $AB = BA$.
(b) A is a square matrix.
(c) B is a square matrix.
(d) Both (a) and (b) are correct.

5. If both U and V are subspaces of a vector space W, then

(a) $U \cap V$ is a subspace of W.
(b) $U + V$ is a subspace of W.
(c) the union need not be a subspace of W.
(d) all of the above.

6. One of the following statements is always true:

(a) If $f,\ g \in C(-\infty, \infty)$ such that $W[f,g](x) = 0$ for all $x \in (-\infty, \infty)$, then f and g are linearly dependent.
(b) A subset S of a vector space V is linearly independent if no element in S can be written as a linear combination of the other elements of S.
(c) Any element in a linearly dependent set can be written as a linear combination of the other elements in the set.
(d) all of the above.

7. The statement *An $n \times n$ matrix A is nonsingular* is equivalent to

(a) $\vec{0} \in N(A)$.
(b) $\text{Span}(\vec{a}_1, ..., \vec{a}_n) = R^n$.
(c) $\text{rank}(A) + \text{nullity}(A) = n$.
(d) all of the above.

8. Which of the following transformations $T : R^2 \longrightarrow R^2$ is not linear?

(a) Reflection: $T(\vec{x}) = -\vec{x}$.
(b) Contraction: $T(\vec{x}) = \alpha\vec{x}, 0 < \alpha < 1$.
(c) Expansion: $T(\vec{x}) = \alpha\vec{x}, 1 < \alpha$.
(d) Translation: $T(\vec{x}) = \vec{x} + \vec{a}$

9. In R^2 with $||\vec{x}||_1 = |x_1| + |x_2|$, $||\vec{x}||_2 = \sqrt{x_1^2 + x_2^2}$, and $||\vec{x}||_\infty = \max\{|x_1|, |x_2|\}$,

(a) $||\vec{x}||_1 \le ||\vec{x}||_2 \le ||\vec{x}||_\infty$.
(b) $||\vec{x}||_1 \le ||\vec{x}||_\infty \le ||\vec{x}||_2$.
(c) $||\vec{x}||_\infty \le ||\vec{x}||_1 \le ||\vec{x}||_2$.
(d) $||\vec{x}||_\infty \le ||\vec{x}||_2 \le ||\vec{x}||_1$.

Math. Dept.

Math 434: Advanced Linear Algebra

M. Saleh

LA

First Semester 2012/2013

Final Exam

Student Name:______________ Number:__________ Section __

1. (a) Prove that the eigenvectors corresponding to distinct eigenvalues are linearly independent
   (b) Prove that the the generalized eigenvectors in a chain are linearly independent
   (c) Let $L : C^2 \to C^2$ be a linear operator defined by $L(z, w) = (z + w, z - w)$. Find a basis for C^2 so that the matrix representation of L with respect to this basis is upper triangular.
   (d) Show that an orthogonal set of vectors is linearly independent
   (e) Show that a real symmetric matrix is diagonalizable
2. (a) Show that any linear operator over a finite dimensional complex space has an eigenvector
   (b) Show that any linear operator over a finite dimensional complex space has an upper triangular matrix representation
   (c) Define an inner product on R^n by $< x, y >= x^t y, x, y \in R^n$. Let A be an $n \times n$ matrix. Show that $||Ax||^2 =< x, A^T Ax >$, for every $x \in R^n$.
   (d) Let λ_1, λ_2 be distinct eigenvalues of A. Let x be an eigenvector of A corresponding to λ_1, y be an eigenvector of A^t corresponding to λ_2. Show that x, y are orthogonal vectors.

$A \quad x \longrightarrow \lambda_1$
$A^t \quad y \longrightarrow \lambda_2$

$\beta(\lambda_1 - \lambda_k$

3. (a) Use Gram-Schmidt to find the distance between the point $(1, 1, 1)$ and the plane $x - y = 1$
   (b) Use Gram-Schmidt to find the formula for the distance between the point (x_0, y_0) and the line $ax + by = c$
   (c) Use Gram-Schmidt to approximate $sinx$ by a polynomial of degree 1. (use inner product on P_n to be $< f, g >= \int_0^1 fg dx$
   (d) Find the adjoint of the linear operator $L : R^3 \to R^2$ defined by $L(x, y, z) = (x - y, x + z - y)$ with respect to the standard basis of R^3 and the basis $(1, 1), (1, 2)$ for R^2

10. If B is similar to A, then

(a) $\det(B) = \det(A)$.

(b) $\operatorname{tr}(B) = \operatorname{tr}(A)$.

(c) $\operatorname{rank}(B) = \operatorname{rank}(A)$.

(d) all of the above.

11. An $n \times n$ matrix A will never have a zero eigenvalue if

(a) $A^t = A$.

(b) $A^t = -A$.

(c) $A^t = A^{-1}$.

(d) all of the above.

Q#1 (20%) Which of the following ... which is false

T 1) If A is singular nxn matrix then $\lambda=0$ is an eigenvalue of A

F 2) If an $n\times n$ matrix A is diagonalizable then A has n distinct eigenvalues

F 3) If S, T are subspaces of a vector space V then $S\cup T$ is also a subspace of V

T 4) If X, Y are two eigenvectors belonging to the same eigenvalue λ then X and Y are linearly independent

T 5) If $\{v_1, v_2, v_3\}$ is a basis for the vector space R^3 then $\{v_1, v_1+v_2, v_1+v_2+v_3\}$ form a basis for R^3

T 6) If $L: R^4 \to R^3$ is a linear transformation then $\ker(L) \neq \{0\}$.

T 7) If λ_1, λ_2 are two distinct eigenvalues for A and X_1, X_2 are corresponding eigenvectors for λ_1, λ_2 respectively then X_1, X_2 are linearly independent

F 8) If A is an $n\times n$ matrix and $Ax=b$ has more than one solution for some $b\in R^n$ then rank(A)=n

T 9) If V is an n dimensional vector space and W_1, W_2 are two subspaces of V and $W_1 \neq W_2$ then $\dim(W_1) \neq \dim(W_2)$.

T 10) Let A be 2×2 matrix such that $\det(A)=5$ and Trace(A)=6 then A is diagonalizable

T 11) Let A be $n\times n$ nonsingular matrix and λ is an eigenvalue of A then $\frac{1}{\lambda}$ is an eigenvalue of A^{-1}

T 12) If V is an n dimensional vector space then any n linearly independent set of vectors in V is a basis for V

T 13) Let A be an $m\times n$ matrix then if AX=b is consistant for every $b\in R^m$ then $m\leq n$

T 14) The dimension of the subspace $W=\{A\in R^{2\times 2} : A \text{ is diagonal}\}$ is 2.

F 15) $\{(1,1,2)^T, (2,1,1)^T, (4,3,5)^T, (1,0,-1)^T\}$ is a spanning set for R^3

F 16) If A is an nxn nonsingular matrix then A is diagonalizable.

T 17) Let ... defined by L(x,y,z)=(x-2y, 5x+y+2) then L is a linear transformation

18) T... matrices is diagonal.

... consistent for every vector b in R^n

... vectors in V span the vector space V.

Q#2 (44%) Circle the correct answer

1- One of the following is a subspace of R^3

a- $W_1 = \{(a, b, ab) ; a, b \in R\}$ b- $W_2 = \{(a, b, a-b) | a, b \in R\}$

c- $W_3 = \{(1, b, c) / b, c \in R\}$ d- $W_4 = \{(a-b, b, 2a+1) | a, b \in R\}$

2- If $W = \{p(x) \in P_3 ; p(1) = 0\}$ then $\dim W =$

a) 1 b) 2

c) 3 d) 4

3) One of the following is a basis of R^3

a) $\{(1, 2, 3), (2, 5, 2), (1, 3, -1)\}$

b) $\{(1, 2, 3), (2, 5, 2), (0, 0, 0)\}$

c) $\{(1, 2, 3), (2, 5, 2)\}$

d) $\{(1, 2, 3), (0, 5, 2), (0, 0, 4)\}$

4) Let $L : R^2 \to R^2$ be a linear transformation defined by $L(x, y) = (x+2y, x+4y)$ then the matrix representing L with respect to the standard basis is

a) $\begin{bmatrix} 1 & -2 \\ -1 & 4 \end{bmatrix}$ b) $\begin{bmatrix} 1 & -2 \\ 1 & 4 \end{bmatrix}$

c) $\begin{bmatrix} 1 & 2 \\ 1 & 4 \end{bmatrix}$ d) $\begin{bmatrix} -2 & 1 \\ 4 & 1 \end{bmatrix}$

5) Let $L : R^2 \to R^3$ be a linear transformation defined by $L(1,0)^t = (2,1,0)^t$, $L(0,1)^t = (5,0,4)^t$, then $L(3,2)^t =$

a) $(3, 2, -1)^t$ b) $(16, 3, 8)^t$

c) [illegible] d) $(11, 3, 8)^t$

[illegible] with $|A| = 6$, $|B| = 12$ then $|2(A^{-1}B)| =$

b) 16

c) $\frac{1}{2}$

7. One of the following is a subspace of $\mathbb{R}^{2\times2}$

a) All diagonal 2×2 matrices

b) All elementary 2×2 matrices

c) All nonsingular 2×2 matrices

d) All nonzero 2×2 matrices

8. Let $L:\mathbb{R}^3 \to \mathbb{R}^3$ be a linear transformation defined by $L\begin{pmatrix}x_1\\x_2\\x_3\end{pmatrix}=\begin{pmatrix}0\\x_1-x_1\\x_1-x_2\end{pmatrix}$ then $\ker(L)$ is

a) $\{(\alpha,\alpha,\alpha):\alpha\in\mathbb{R}\}$

b) $\{(\alpha,0,\alpha):\alpha\in\mathbb{R}\}$

c) $\{(0,\alpha,\alpha):\alpha\in\mathbb{R}\}$

d) $\{(\alpha,0,0):\alpha\in\mathbb{R}\}$

9. The coordinates of the vector $p(x)=2x^2-5x+2$ in the ordered basis $F=\{[1],[-x],[1-x^2]\}$

a) $(-2,-1,5)^T$

b) $(1,2,5)^T$

c) $(5,-2,-1)^T$

d) $(-1,5,-2)^T$

10. Let $L:\mathbb{R}^3 \to \mathbb{R}^3$ be a linear transformation defined by $L\begin{pmatrix}x_1\\x_2\\x_3\end{pmatrix}=\begin{pmatrix}x_1-x_1\\0\\x_3\end{pmatrix}$ then dimension of range(L) is

a) 1

b) 2

c) 3

d) 0

11. Let A be 3×3 matrix such that $tr(A)=1$, and $\lambda_1=2$, $\lambda_2=-1$ be eigenvalues of A then

a) A is nonsingular

b) $\det(A)=-2$

c) A is diagonalizable

d) $\det(A)=2$

12. ... matrix such that $\lambda=2$ is an eigenvalue of A

... equal to 2

13- Let A be 3×3 matrix such that rank(A)=3 then

a) A is nonsingular

b) $AX=0$ has the trivial solution only

c) A is diagonalizable

d) a and b are true

14- The vectors $\{\sin^2 x, 1, \cos^2 x\}$ in $C[0,2\pi]$ are

a) Linearly independent

b) Span the vector space $C[0,2\pi]$

c) Form a basis for the vector space $C[0,2\pi]$

d) None of the above

15) If $A=\begin{bmatrix}1 & 4-a & 5\\ 1 & a & 7-a\\ 0 & 0 & a-2\end{bmatrix}$ then the values of a for which rank(A)=2 is

a) $a=1$

b) $a\neq2$

c) $a=-1$

d) there is no value for a

16) If $A=\begin{bmatrix}0 & 2 & 5\\ 0 & 2 & 4\\ 0 & 0 & 1\end{bmatrix}$ then

a) nullity of A=2

b) A is in row echelon form

c) A is diagonalizable

d) All of the above

17) If $\lambda=0$ is an eigenvalue of an $n\times n$ matrix A then

a) Column space of A is R^n

b) Row vectors of A are linearly independent

c) Null space of A is {0}

d) Nullity of A > 0

a) $\{x, x^2 - x\}$

b) $\{x^2 - x\}$

d) $\{x^2 - 1\}$

19) Let V be a vector space with dimension(V)=4 then

a) Any four vectors in V are linearly independent

b) Any spanning set of four vectors form a basis for V

c) Any four vectors in V span V

d) None of the above

20) If v_1, v_2, v_3 are linearly independent vectors in a vector space V and if $v \in V$ does not belong to span$\{v_1, v_2, v_3\}$ then

a) $\{v_1, v_2, v_3, v\}$ is linearly dependent

b) $\{v_1, v_2, v_3, v\}$ is a spanning set for V

c) $\{v_1, v_2, v_3, v\}$ is linearly independent

d) dimension of V ≤ 3

21- Let $V = P_3$. Let $E = \{2, 1+2x, 1+x+5x^2\}$, $F = \{x, 1, x^2\}$ be two basis, then the matrix correspond to change of basis from E to F is

a- $\begin{bmatrix} 2 & 1 & 1 \\ 0 & 2 & 1 \\ 0 & 0 & 5 \end{bmatrix}$ b- $\begin{bmatrix} 0 & 2 & 1 \\ 2 & 1 & 1 \\ 0 & 0 & 5 \end{bmatrix}$ c- $\begin{bmatrix} 0 & 2 & 1 \\ 2 & 1 & 1 \\ 1 & 0 & 5 \end{bmatrix}$ d- $\begin{bmatrix} 2 & 1 & 1 \\ 0 & 2 & 1 \\ 0 & 1 & 5 \end{bmatrix}$

22) the null space of $A = \begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 1 \\ 1 & 2 & 2 \end{bmatrix}$ is

a) $\{(0,0,\beta)^T \mid \beta \in R\}$

b) $\{(0,0,0)^T\}$

c) $\{(0,-\beta,\beta)^T \mid \beta \in R\}$

d) $\{(\beta,0,-\beta)^T \mid \beta \in R\}$

18) Let $S=\{p(x)\in P_3 \,/\, p(1)=0 \text{ and } p(0)=0\}$ then a basis for S is

a) $\{x, x^2-x\}$

b) $\{x-1, x^2-x\}$

b) $\{x^2-x\}$

d) $\{x^2-1\}$

19) Let V be a vector space with dimension(V)=4 then

a) Any four vectors in V are linearly independent

b) Any spanning set of four vectors form a basis for V

c) Any four vectors in V span V

d) None of the above

20) If v_1, v_2, v_3 are linearly independent vectors in a vector space V and if $v\in V$ does not belong to span$\{v_1, v_2, v_3\}$ then

a) $\{v_1, v_2, v_3, v\}$ is linearly dependent

b) $\{v_1, v_2, v_3, v\}$ is a spanning set for V

c) $\{v_1, v_2, v_3, v\}$ is linearly independent

d) dimension of V ≤ 3

21- Let $V=P_3$ Let $E=\{2, 1+2x, 1+x+5x^2\}$, $F=\{x, 1, x^2\}$ be two basis, then the matrix correspond to change of basis from E to F is

a- $\begin{bmatrix} 2 & 1 & 1 \\ 0 & 2 & 1 \\ 0 & 0 & 5 \end{bmatrix}$ b- $\begin{bmatrix} 0 & 2 & 1 \\ 2 & 1 & 1 \\ 0 & 0 & 5 \end{bmatrix}$ c- $\begin{bmatrix} 0 & 2 & 1 \\ 2 & 1 & 1 \\ 1 & 0 & 5 \end{bmatrix}$ d- $\begin{bmatrix} 2 & 1 & 1 \\ 0 & 2 & 1 \\ 0 & 1 & 5 \end{bmatrix}$

22) the null space of $A=\begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 1 \\ 1 & 2 & 2 \end{bmatrix}$ is

a) $\{(0,0,\beta)^T \,/\, \beta\in R\}$

b) $\{(0,0,0)^T\}$

c) $\{(0,-\beta,\beta)^T \,/\, \beta\in R\}$

d) $\{(\beta,0,-\beta)^T \,/\, \beta\in R\}$

Question#3 (12%) Let $L: P_3 \to P_3$ be a linear transformation defined by $L(p(x)) = p'(x) + p(0)$, let $E=\{1+x, -1+x, x^2\}$, $F=\{1, 1-x, x^2\}$, be two bases for P_3 $\{1+x, -1+x, x^2\}$ and $F=\{1, 1-x, x^2\}$ respecti

a) Find the matrix of L with respect to the basis $E=\{1+x, -1+x, x^2\}$

$$\begin{bmatrix} 2 & 0 & 2 \\ 0 & 0 & -2 \\ 0 & 0 & 0 \end{bmatrix}$$

b) find the coordinate vector of $p(x)=2x^2+2x$ in the basis E

c) Use the matrix in part (a) to find coordinate vector of $L(p(x)=2x^2+2x)$ in the basis F

(4) The LU decomposition of the matrix $\begin{bmatrix} 2 & 4 & 2 \\ 1 & 5 & 2 \\ 4 & -1 & 9 \end{bmatrix}$ is $\sim \begin{bmatrix} 2 & 4 & 2 \\ 0 & 3 & 1 \\ 0 & -9 & 5 \end{bmatrix} \sim \begin{bmatrix} 2 & 4 & 2 \\ 0 & 3 & 1 \\ 0 & 0 & 8 \end{bmatrix} = U$

$L = \begin{bmatrix} 1 & 0 & 0 \\ \frac{1}{2} & 1 & 0 \\ 2 & -3 & 1 \end{bmatrix}$

(a) $L = \begin{bmatrix} 1 & 0 & 0 \\ -\frac{1}{2} & 1 & 0 \\ -2 & -3 & 1 \end{bmatrix}, U = \begin{bmatrix} 2 & 4 & 2 \\ 0 & 3 & 1 \\ 0 & 0 & 8 \end{bmatrix}$

(b) $L = \begin{bmatrix} 1 & 0 & 0 \\ \frac{1}{2} & 1 & 0 \\ 2 & [illegible] & 1 \end{bmatrix}, U = \begin{bmatrix} 2 & 4 & 2 \\ 0 & 3 & 1 \\ 0 & 0 & 8 \end{bmatrix}$

(c) $L = \begin{bmatrix} 1 & 0 & 0 \\ -\frac{1}{2} & 1 & 0 \\ 2 & 3 & 1 \end{bmatrix}, U = \begin{bmatrix} 2 & 4 & 2 \\ 0 & 3 & 1 \\ 0 & 0 & 8 \end{bmatrix}$

(d) None

(5) One of the following sets is a subspace of P_4

non empty

(a) $\{f(x) \in P_4 : f(0) = 1\}$

$(g+f)(1) = g(1) + f(1) = 0 + 0 = 0$

(b) $\{f(x) \in P_4 : f(1) = 1\}$

$(\alpha f)(1) = \alpha f(1) = (\alpha)(0) = 0$

(c) $\{f(x) \in P_4 : f(1) = 0\}$

(d) $\{f(x) \in P_4 : f(x) = x^3 + bx^2 + cx, b, c \in R\}$

(6) The Rank of $A = \begin{bmatrix} 1 & 2 & 3 & 0 \\ 0 & -1 & 1 & 0 \\ 2 & 4 & 0 & 1 \end{bmatrix}$ is $\sim \begin{bmatrix} 1 & 2 & 3 & 0 \\ 0 & 1 & -1 & 0 \\ 0 & 0 & -6 & 1 \end{bmatrix} \sim \begin{bmatrix} 1 & 2 & 3 & 0 \\ 0 & 1 & -1 & 0 \\ 0 & 0 & 1 & -1/6 \end{bmatrix}$

(a) 1

(b) 2

(c) 3

(d) 4

(7) If A is a nonzero $n \times m$ matrix then

(a) $Rank(A) \geq \min(n, m)$

(b) $Rank(A) \geq$ [illegible]

(c) [illegible]

(d) $0 \leq$ [illegible]

(8) [illegible]

(a) [illegible]

(b) [illegible]

(c) ~~4~~

(d) ~~0~~

(9) If a 4×4 matrix A has $1, -1, 3, 5$ as its eigenvalues then $det(A) =$

(a) 8

(b) -15

(c) 15

(d) -8

$det(A) = (1)(-1)(3)(5) = -15$

(10) Let $A = \begin{bmatrix} -1 & 3 \\ 0 & 2 \end{bmatrix}$ then the eigenvalues of A^{100} are

(a) $1, 2$

(b) $-1, 2$

(c) $1, 2^5$

(d) *None*

(11) If the coefficient matrix of the linear system $AX = b$, for any $b \in R^3$ is

$$A = \begin{bmatrix} 1 & 2 & 3 & 0 \\ 0 & -1 & 1 & 0 \\ 2 & 4 & 0 & 1 \end{bmatrix} \text{ then}$$

(a) The system is consistent

(b) The system is inconsistent

(c) The system has a unique solution

(d) None

(12) If A an 3×3 matrix such that $AX = 0$ for a nonzero X, then one of the following might be the characteristic polynomial of A.

(a) $x^5 - 3x$

(b) $x^2 - 3x$

(c) $x^3 - 3x + 2$

(d) $x^3 - 3x$

(13) If A is an $n\times n$ nonsingular matrix then

(a) $N(A) = \{0\}$

(b) The rows and columns of A are linearly independent

(c) $Rank(A) = n$

(d) All of the above

(14) If A is a 4×4 matrix such that 2 is the only eigenvalue of A, then the characteristic polynomial of A is

(a) $x - 2$

(b) [illegible]

(24) Let $U = \{(x,y) / \ y = x + 1\}$. Then U is a subspace of R^2.

(25) If A is an $n \times n$ matrix and has n linearly independent eigenvectors, then A is nonsingular

~~(26) If $rank(A) = rank(A|B)$ then the linear system $AX = B$ is consistent.~~

~~(27) If Wronskain$(f_1, \ldots, f_n) = 0$ then $f_1, \ldots, f_n$ are linearly dependent~~

(28) If A is a 4×4 matrix such that $N(A) = 0$ then A is nonsingular.

(29) If V is a vector space such that $dim(V) = 4$ and $v_1, v_2, \ldots, v_4$ are distinct vectors in V, then $Span\{v_1, v_2, \ldots, v_4\} = V$.

False (30) If V is a vector space such that $Span\{v_1, v_2, \ldots, v_4\} = V$, then $v_1, v_2, \ldots, v_4$ are linearly independent.

(31) Similar matrices have the same eigenvectors.

~~(32) Any singular matrix is defective (not diagonalizable).~~

~~(33) Any matrix with a zero eigenvalue is singular.~~

~~(34) Any triangular matrix is diagonalizable.~~

~~(35) If an $n \times n$ matrix A is diagonalizable then A must have n linearly independent~~ eigenvectors

True (36) If A is an $n \times n$ matrix diagonalizable then $rank(A) = n$.

(37) If A, B are square $n \times n$ matrices, then $(A + B)(A - B) = A^2 - B^2$. $AB + BA$ $\boxed{AB \neq BA}$

(38) If A is a 4×7 matrix with $Rank(A) = 4$, then the homogeneous system $AX = 0$ has a nontrivial solution.

False (39) If A is an $n \times n$ symmetric matrix, then $Rank(A) = n$.

(40) Every set of vectors spanning R^3 contains at least 3 vectors.

(41) If S is a subset of a vector space V that doesn't contain the zero vector, then S is not a subspace of V.

(42) The set $S = \{v_1, \ldots, v_n\}$ is a spanning set of a vector space V if every vector in V is a linear combination of the set S.

False (43) The transition matrix of two basis could be singular.

(44) If $v_1, v_2, \ldots, v_n$ span a vector space V and v_1 is a linear combination of $v_2, \ldots, v_n$, then $V = Span\{v_1, v_2, v_3\} = V$.

(45) If two nonzero vectors in a vector space V are linearly dependent, then each one of them is a scalar multiple of the other.

(46) The vectors $1, x, x - 1$ are linearly dependent.

(47) If 3 vectors span a vector space V, then a collection of 6 vectors in V span V.

False (48) If A is an $m \times n$ matrix, the rows of A spans R^n.

(49) The coordinate vector of $2 + 6x$ with respect to the basis $[2x, 4]$ is $(2, 3)^t$

False (50) If V is a vector space with dimension $n > 0$, then any set of n or more vectors in V are linearly dependent.

False (51) If two matrices are row equivalent, the[illegible]

False (52) If b is in the column [illegible]

True (53) If L is a linear transf[illegible] then $L(0_V) = 0_W$

Q1 (10 points) True or False

(1) If A is $n \times n$ and diagonalizable, then A has n different eigenvalues

(2) A homogeneous system is always consistent.

(3) If $(A|B) = \left(\begin{array}{ccc|c} 1 & 1 & 2 & 4 \\ 2 & -1 & 2 & 6 \\ 0 & 0 & 0 & 1 \end{array}\right)$ is the Augmented matrix of the system $AX = B$ then the system has no solution

(4) Any nonsingular matrix is diagonalizable

(5) If A, B are two square matrices and $AB = 0$ then A and B are singular

(6) Let A be a square and nonsingular $n \times n$ matrix. If $|adjA| = |A|$ then A is 2×2

(7) If A is $n \times n$ and has n linearly independent eigenvectors, then A is nonsingular

(8) If $rank(A) = rank(A|B)$ then the linear system $AX = B$ is consistent.

(9) If Wronskain$(f_1, \ldots, f_n) = 0$ then $f_1, \ldots, f_n$ are linearly dependent

(10) If x_0 is a solution of the nonhomogeneous system $AX = B$ and x is a solution of the homogeneous system $AX = 0$. Then $x + x_0$ is a solution of the nonhomogeneous system $AX = B$.

(11) If A is symmetric and skew symmetric then $A = 0$. (A is skew symmetric if $A = -A^T$).

(12) Similar matrices have the same eigenvectors.

(13) Any singular matrix is defective.

(14) Any matrix with a zero eigenvalue is defective.

(15) Any triangular matrix is diagonalizable.

(16) If an $n \times n$ matrix A is diagonalizable then A must have n linearly independent eigenvectors

~~(17) If A is diagonalizable then $rank(A) = n$.~~

(18) If A is a square matrix, then A and A^T must have the same eigenvalues.

(19) If A is an $n \times n$ diagonalizable matrix, then A has n different eigenvalues

(20) If A is a 3×3 matrix such that $det(A) = 2$, then $det(3A) = 6$

(21) If $A = \begin{pmatrix} 1 & 2 & 3 & 0 \\ 0 & -1 & 1 & 2 \\ 2 & 3 & 7 & 2 \end{pmatrix}$ is the coefficient matrix of the system $AX = b$, for every $b \in R^3$ then the system has infinitely many solutions

(22) Any diagonalizable matrix is nonsingular

(23) If A is a 3×7 matrix then it ... of the column space of A to be 6

(c) Find a basis for Range L

linearly independent $\rightarrow$ basis for $\mathbb{R}^2$

range $= \mathbb{R}^2$

basis $= \{\vec{e}_1, \vec{e}_2\}$

$$p(x) = ax^2 + bx + c$$

$$\int_0^1 p(x)\,dx = \frac{ax^3}{3} + \frac{bx^2}{2} + cx \Big]_0^1$$

$$= \frac{a}{3} + \frac{b}{2} + c$$

$$L(\vec{x}) = \begin{bmatrix} \frac{a}{3} + \frac{b}{2} + c \\ c \end{bmatrix} = a \begin{bmatrix} \frac{1}{3} \\ 0 \end{bmatrix} + b \begin{bmatrix} \frac{1}{2} \\ 0 \end{bmatrix}$$

(18) A basis for the vector space spanned by $1-x-x^2$, $1+x+x^2$, $2-x$, $1-x$ from this set of vectors is

(a) $1-x-x^2, 1+x+x^2, 2-x$
(b) $1-x-x^2, 1+x+x^2$
(c) $1-x-x^2, 1+x+x^2, 2-x, 1-x$
(d) $1-x-x^2, 1-x$
(e) $1-x-x^2, 2-x$

$3-1=2 \Rightarrow$ dependent

(19) The dimension of the null space of $\begin{pmatrix} 1 & 1 & 2 & 1 & 4 \\ 2 & -1 & 2 & -1 & 6 \\ 3 & 0 & 4 & 0 & 10 \end{pmatrix}$ is

(a) 0
(b) 1
(c) 2
(d) 3
(e) 4

3 free variables

20) A basis for the row space of $\begin{pmatrix} 1 & 1 & 2 & 1 & 4 \\ 2 & -1 & 2 & -1 & 6 \\ 3 & 0 & 4 & 0 & 10 \end{pmatrix}$ is

(a) $(1,1,2,1,4), (2,-1,2,-1,6)$
(b) $(1,1,2,1,4), (2,-1,2,-1,6), (3,0,4,0,10)$
(c) $(1,1,2,1,4)$
(d) $(1,2,3), (1,-1,0)$
(e) $(1,1,2,1,4), (0,-2,-2,-3,-2), (0,0,0,0,0)$

2. **(19 points) Circle the most correct answer**

(1) Let A be nonsingular. Then
(a) If A is symmetric then A^{-1} is symmetric $A^{-1} = (A^T)^{-1} = (A^{-1})^T \Rightarrow$ symmetric
(b) If A is triangular then A^{-1} is triangular
(c) If A is diagonalizable then A^{-1} is diagonalizable
(d) All of the above

(2) If A is a 4×3 matrix such that $N(A) = 0$, and $b = \begin{pmatrix} 0 \\ 3 \\ 2 \\ 1 \end{pmatrix}$ $\in \mathbb{R}^4 \Rightarrow$ consistent, then

nonsingular

(a) It is possible that $AX = b$ has infinitely many solutions
(b) The system $AX = b$ [illegible]
(c) [illegible]
(d) [illegible]

(d) Every basis for V is contained in a basis for W.
(e) None

(13) For any finite n-dimensional vector space V with a basis B, [illegible]
(a) The coordinate vector of any vector v in V is in R^n.
(b) A subspace of V is a subset of V that contains a zero vector and is closed under the operation of addition
(c) If V is a nonempty subset of a vector space W, and the operations defined on V coincide with the operations defined on W, then V is a subspace of W [illegible]
(d) The [illegible]
(e) None

(14) For any vector space V [illegible] then V is a subspace of R^n for some positive [illegible]
(a) If V is finite-dimensional, then every infinite subset of V is linearly [illegible]
(b) If V is infinite-dimensional, then every [illegible] independent
(c) If V is a nonempty subset of a vector space W, and the operations defined on W [illegible] coincide with the operations defined on W, then [illegible] infinite subset of V is linearly [illegible]
(d) If V is finite-dimensional, then an infinite subset of V is linearly dependent
(e) None

(15) One of the following is not a linear transformation
(a) $T: R^2 \to R^2$ defined by $T(x, y) =$ [illegible]
(b) $T: R^2 \to R^2$ defined by $T(x, y) =$ [illegible]
(c) $T: R^2 \to R^2$ defined by $T(x, y) =$ [illegible]
(d) $T: R^2 \to R^3$ defined by $T(x, y) = (0, 0, 0)$
(e) $T: R^2 \to R^2$ defined by $T(x, y) =$ [illegible]

(16) One of the following linear transformations is one-to-one [illegible]
(a) $T: R^2 \to R^2$ defined by $T(x, y) =$ [illegible]
(b) $T: R^2 \to R^2$ defined by $T(x, y) =$ [illegible]
(c) $T: R^2 \to R^2$ defined by $T(x, y) =$ [illegible]
(d) $T: R^2 \to R^2$ defined by $T(x, y) =$ [illegible]
(e) $T: R^2 \to R^2$ defined by $T(x, y) =$ [illegible]

(17) The dimension of the subspace [illegible]
[illegible]
(a) 0
(b) 1
[illegible]

(a) $\begin{pmatrix} 2 & -1 \\ -1 & 1 \end{pmatrix}$

(b) $\begin{pmatrix} 2 & -1 \\ 0 & -1 \end{pmatrix}$

(c) $\begin{pmatrix} -1 & 2 \\ -1 & 0 \end{pmatrix}$

(d) $\begin{pmatrix} -1/2 & 0 \\ -1/2 & 1 \end{pmatrix}$

(e) *None*

(9) Let A be an arbitrary $n \times n$ matrix. Then the rank of A

(a) equals the dimension of the column space of A

(b) equals the dimension of the null space of A

(c) equals n

(d) equals the determinant of A

(e) None

(10) Let A be an arbitrary $n \times n$ matrix. Then

(a) The row space of A equals the column space of A

(b) The row space of A equals the null space of A

(c) The row space of A is contained in the column space

(d) The row space of A has the same dimension as the column space of A

(e) None

(11) Let u and v be vectors in R^n, and let B be a basis for R^n. Then

(a) the coordinate vector of u with respect to B never equals u

(b) the coordinate vector of v with respect to B equals v

(c) the coordinate vector of $u + v$ with respect to B need not equal the sum of the coordinate vector of u and the coordinate vector of v with respect to B.

(d) u and v are equal if their coordinate vectors with respect to B are equal

(e) None

(12) Let [illegible] dim V. Then [illegible] need not be equal

(a) [illegible]

(b) [illegible]

(c) [illegible]

(b) $rank(A) = n$

(c) $Nullity(A) = \{0\}$

(d) All of the above

(2) Suppose that $T : V \to W$ is a linear transformation whose 2×2 standard matrix A, and $rank(A) = 2$. Then

(a) Range $T = W$

(b) $Ker T = \{0\}$

(c) $nullity(A) = \{0\}$

(d) All the above?

(3) An $n \times n$ matrix A is invertible if

(a) The columns of A are linearly independant

(b) The columns of A span R^n

(c) The rows of A are linearly independant

(d) $nullity(A) = 0$

(e) all of the above

(4) Let S be a finite subset of a subspace W of R^n. Then S is a basis for W if

(a) S is linearly independent

(b) S spans W

(c) the number of vectors in S equals the dimension of W

(d) every vector in W is a linear combination of vectors in S

(e) None

(5) Suppose that W is a subspace of R^n. Then

(a) the dimension of W is greater than n

(b) every basis of R^n contains a basis of W

(c) every linearly independent subset of W has at most n vectors

(d) the dimension of W equals n

(e) None

(6) One of the following linear transformations is onto

(a) $T : R^3 \to R^3$ defined by $T(x, y, z) = (x, y - z, z)$ $= x(1,0,0) + y(0,1,0) + z(0,-1,1)$

(b) $T : R^3 \to R^3$ defined by $T(x, y, z) = (x - y, y, x - y)$ $\begin{vmatrix} 1 & 0 & 0 \\ 0 & 1 & -1 \\ 0 & 0 & 1 \end{vmatrix} = 1 \Rightarrow$ nonsingular $\Rightarrow$ independent $\Rightarrow$ basis

(c) $T : R^2 \to R^3$ defined by $T(x, y) = (x, y, x - y)$

(d) $T : R^2 \to R^3$ defined by $T(x, y) = (x, y, x)$

(e) $T : R^2 \to R^3$ defined by $T(x, y) = ($[illegible]$)$

(7) Let $T : R^3 \to R^2$ be a linear [illegible]

$T(e_1) = (1, 2, -1)$, $T(e_2)$ [illegible]

(a) $(2, 0,$ [illegible]

Question#4 (12%) Let $L : R^3 \to R^2$ be a linear transformation defined by $L\left(\begin{bmatrix} x \\ y \\ z \end{bmatrix}\right) = \begin{bmatrix} x - y + z \\ 2x - 4z \end{bmatrix}$

a) Find Ker(L)

$$\begin{pmatrix} 1 & 2 \\ -1 & 0 \\ 1 & -4 \end{pmatrix}\begin{pmatrix} a \\ b \end{pmatrix} = 0$$

$a + 2b = 0$ $\qquad a = 0$

$-a = 0$ $\qquad b = 0$

$a - 4b = 0$

Ker(L) = 0

b) Find basis and dimension for Ker(L)

Dimension = 1

~~Basis~~

c) Find range of L

~~Range(L)~~